第一作者

项目来

病态矩阵

零

模型才会发生病态性问题

出采用主成分估计法来求解

GM

数来解决

色系统理论后就一直有学者在分析和讨论

来解决

GM

例。

低信息矩阵的条件数

比较深入

性来源于模型本身

色模型都存在严重的病态性问题

参数的求解以及模型的预测精度

原理，数学推导后

文献

中图法分类号:

灰色系

关键词:

方法

件数

唐利民，

(国家自然科学基金资助项目

指出

14JJ3099)

调整计量单位不会影响灰

文献

410004

文献标

志码

A

1.2

1GM(1,1)

\begin{array}{c}
X^{(0)} = \\
X^{(3)} = \\
\vdots \\
X^{(n)} = \\
\end{array}

\begin{array}{c}
X^{(2)} \\
X^{(5)} \\
\vdots \\
X^{(n)} \\
\end{array}

\begin{array}{c}
- z^{(2)} \\
- z^{(5)} \\
\vdots \\
- z^{(n)} \\
\end{array}

a = [a, b]^T \quad GM(1,1)

1\, \text{年}

\[ Y = \begin{bmatrix}
X^{(2)} \\
X^{(3)} \\
\vdots \\
X^{(n)} \\
\end{bmatrix}, \quad B = \begin{bmatrix}
- z^{(2)} & 1 \\
- z^{(3)} & 1 \\
\vdots & \vdots \\
- z^{(n)} & 1 \\
\end{bmatrix} \tag{1}
\]

\[ a = [a, b]^T = (B^T B)^{-1} B^T Y \tag{2} \]

\[ B^T B = \begin{bmatrix}
\sum_{k=1}^{n} [z^{(1)}(k)]^2 & - \sum_{k=2}^{n} z^{(1)}(k) \\
- \sum_{k=2}^{n} z^{(1)}(k) & n-1
\end{bmatrix} \tag{3} \]

\[ [4, 3] \quad \text{z}^{(1)}(k) > 0 \]

\[ 1-7 \quad 1 \]
系统研究的是小样本情况下的异常很大的问题。

接(17)08([近似为数)

从而得出

关键的条件数取决于文献[8]10(1)。文献[8]10(1)的条件数为

当k=0时条件数高达

当k=0时条件数为

由于X10(1)、X10(2)、...、X10(n)的条件数在某些情况下可能非常大，因此在求解时需要考虑条件数的影响。
Thus, the accuracy

average residual and prediction accuracy

\[ x^{(1)}(k) = \frac{1}{10} \sum_{k=1}^{n} x^{(1)}(k) / 10 \]

\[ E = \frac{1}{10} \sum_{k=1}^{n} (\frac{1}{10} x^{(1)}(k)) \]

\[ F = \frac{1}{10} \sum_{k=1}^{n} (\frac{1}{10} x^{(1)}(k)) \]

\[ a_{10} = \frac{C_{10}D_{10} - (n-1)E_{10}}{(n-1)F_{10} - C_{10}} = \frac{1}{10} C \times \frac{1}{10} D - (n-1) \frac{1}{10} E = (n-1) \frac{1}{10} F - \frac{1}{10} E \]

\[ b_{10} = \frac{D_{10}E_{10} - C_{10}E_{10}}{(n-1)F_{10} - C_{10}} = \frac{1}{10} D \times \frac{1}{10} F - \frac{1}{10} C \times \frac{1}{10} E = (n-1) \frac{1}{10} F - \frac{1}{10} E \]

\[ \text{GM}(1, 1) : \]

\[ x_{10}^{(1)}(k) = x_{10}^{(1)}(k-1) + a_{10} z_{10}^{(1)}(k) \]

\[ x^{(1)}(k) = x^{(1)}(k-1) + x^{(1)}(k) / 10 \]

\[ z^{(10)} = \text{mean}(x_{10}^{(1)}) \]

\[ z^{(10)} = 0.5 x_{10}^{(1)}(k) + 0.5 x_{10}^{(1)}(k-1) = (0.5 x^{(1)}(k) + 0.5 x^{(1)}(k-1)) / 10 \]

\[ (5) \]
\[ x_{i0}(k+1) = \frac{1}{10} x(k+1), \quad x_{i0}(k) = \frac{1}{10} x^{(o)}(k) \tag{13} \]

\[ e^{(o)}(k) = \frac{x^{(o)}(k) - x^{(o)}(k)}{\bar{x}^{(o)}} \times 100\% = \frac{1}{10} \frac{x^{(o)}(k) - x^{(o)}(k)}{\bar{x}^{(o)}} \times 100\% = e^{(o)}(k) \tag{14} \]

\[ e^{(o)}(\text{avg}) = \frac{1}{n} \sum_{k=1}^{n} |e^{(o)}(k)| = \frac{1}{n} \sum_{k=1}^{n} |e^{(o)}(k)| = e^{(o)}(\text{avg}) \tag{15} \]

\[ p^{(o)} = (100 - e^{(o)}(\text{avg}))\% = (100 - e^{(o)}(\text{avg}))\% = p^{(o)} \tag{16} \]

<table>
<thead>
<tr>
<th>CBR</th>
<th>GM(1, 1)</th>
<th>CBR</th>
<th>GM(1, 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01 mm</td>
<td>2</td>
<td>0.01 mm</td>
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</tbody>
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<table>
<thead>
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<tr>
<td>1 h</td>
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<tr>
<td>2 h</td>
<td>0.81</td>
<td>0.81</td>
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<tr>
<td>3 h</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td>4 h</td>
<td>0.83</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Adjust Measurement Unit Algorithm for Ill-posed Problem of GM (1,1) Model

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Abstract: Ill-posed problems exist in the gray system GM (1,1) model. Seriously ill-posed information matrices will occur in larger original measured data values. Reasons which cause ill-posed information matrices in GM (1,1) models are analyzed in detail. A method to adjust the measurement unit of the original measured data values to reduce the conditional number of information matrices is put forward. Based on GM (1,1) model theory, we show that an adjustment to the measured data units of measurement does not affect the model relative residuals, average residuals, or prediction accuracy. Numerical experiments and analysis demonstrate that this method of adjusting the measurement unit algorithm for a GM (1,1) model is easy to implement, simple, accurate, and widely applicable.

Key words: GM (1,1) model; ill-posed problem; information matrices; measurement unit

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