



无定向点优化布局的多边交会测量精度分析

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摘要: 多边交会系统利于提高激光跟踪仪坐标测量精度,但测量精度易受测站布局 and 系统参数标定精度的影响。引入球心拟合的无定向点系统参数标定法,避免传统系统参数标定精度受定向点分布的影响,根据无定向点系统参数标定模型推导出多边交会的最佳测站布局——直角正三棱锥,从而保证多边交会测量精度。仿真结果表明,在5 m范围内,球心拟合的无定向点系统参数标定中误差为0.006 4 mm,最佳测站布局下多边交会的点位中误差为0.005 mm。经标准尺长度测量验证和四面体标准器坐标测量验证,优化后多边交会的长度测量中误差为0.003 6 mm,坐标测量中误差为0.005 3 mm。在无定向点系统参数标定和直角三棱锥布局下,激光跟踪仪多边交会能够实现微米级三维坐标测量。

关键词: 无定向点参数标定;测站布局优化;多边交会测量;激光跟踪仪

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激光跟踪仪作为便携式三坐标测量机^[1],在科学装置安装、精密控制网建立以及机床加工检测等工业测量和精密工程测量领域应用广泛^[2-4]。但激光跟踪仪单站测量受测角误差、仪器预热时间、非均匀温度场、反射器等因素影响^[5-7],其单站坐标测量结果无法满足更高精度的需求^[8]。为满足微米级坐标测量需求,多源融合的统一空间网络法^[9]、分时多站光束法^[10]以及激光测距多边法^[11]得以提出,上述方法均是增加冗余观测值,从而提高测点精度,其中多边法测量精度最高。

多边法是多台激光跟踪仪测距交会得到测点坐标,舍弃了测角误差,该方法测点精度满足三坐标测量机校准要求^[12],但测量精度易受测站布局影响^[13-14],因此遗传算法^[15]、蒙特卡罗解析法^[16]用于其网形优化,但上述优化算法容易陷入局部最优,且计算结果的几何意义不明确。此外,多边法需事先标定各跟踪仪原点在全局坐标系下坐标,该标定过程称为系统参数标定。常规系统参数标定方法需要布设若干定向点、标准尺^[17],存在标定过程复杂、参数标定精度易受定向点分布影响等不足。

为了保证多边交会坐标测量的可靠性,本文引入球心拟合的无定向点系统参数标定法,根据

无定向点系统参数标定模型推导最佳测站布局,基于蒙特卡罗原理分析了系统参数标定中误差以及最佳测站布局下多边交会测点中误差,以标准尺、四面体标准器为距离、坐标测量对象,验证了优化后多边交会高精度坐标测量的效果。

1 激光测距多边交会原理

多边交会根据多个已知测站点 T_i 到 p 点的距离 L_i ,基于空间前方交会原理得到 p 点三维坐标^[14]。由于各测站仪器坐标系相互独立,因此需先定义全局坐标系,如图1所示。

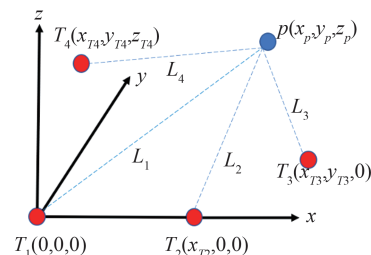


图1 全局坐标系定义

Fig.1 Global Coordinate System Definition

图1中测站 T_1 为原点, T_2 与 T_1 所在直线为 x 轴, T_1 、 T_2 、 T_3 所在平面的法线为 z 轴,根据右手坐标系原则建立 y 轴。各测站 T_i 至 p 点距离为 L_i ,

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当 $i=1, 2 \dots 4$ 时, 其计算式为:

$$L_1 = \sqrt{x_p^2 + y_p^2 + z_p^2} \quad (1a)$$

$$L_2 = \sqrt{(x_p - x_{T2})^2 + y_p^2 + z_p^2} \quad (1b)$$

$$L_3 = \sqrt{(x_p - x_{T3})^2 + (y_p - y_{T3})^2 + z_p^2} \quad (1c)$$

$$L_4 = \sqrt{(x_p - x_{T4})^2 + (y_p - y_{T4})^2 + (z_p - z_{T4})^2} \quad (1d)$$

将式(1)按泰勒级数展开并线性化得:

$$v_i^L = e_i^x \delta x_p + e_i^y \delta y_p + e_i^z \delta z_p - (L_i - L_i^0) \quad (2)$$

式中, v_i^L 为激光跟踪仪测距残差; $(\delta x_p, \delta y_p, \delta z_p)$ 是

p 点坐标改正数; $e_i^x = -\frac{x_{Ti} - x_{p0}}{L_i^0}$, $e_i^y = -\frac{y_{Ti} - y_{p0}}{L_i^0}$,

$e_i^z = -\frac{z_{Ti} - z_{p0}}{L_i^0}$; (x_{p0}, y_{p0}, z_{p0}) 为 p 点近似坐标; $L_i^0 =$

$$\sqrt{(x_{Ti} - x_{p0})^2 + (y_{Ti} - y_{p0})^2 + (z_{Ti} - z_{p0})^2}$$

由式(2)可得测距误差的矩阵形式:

$$V = Ax - l \quad (3)$$

式中, $V = [v_1^L \ v_2^L \ v_3^L \ v_4^L]^T$; $x = [\delta x_p \ \delta y_p \ \delta z_p]^T$;

$$A = \begin{bmatrix} e_1^x & e_1^y & e_1^z \\ e_2^x & e_2^y & e_2^z \\ e_3^x & e_3^y & e_3^z \\ e_4^x & e_4^y & e_4^z \end{bmatrix}; l = \begin{bmatrix} L_1 - L_1^0 \\ L_2 - L_2^0 \\ L_3 - L_3^0 \\ L_4 - L_4^0 \end{bmatrix}$$

根据间接平差最小二乘原理, 式(3)中 p 点坐标改正数为:

$$x = (A^T W A)^{-1} A^T W l = Q_X A^T W l \quad (4)$$

式中, $Q_X = (A^T W A)^{-1} = \begin{bmatrix} q_{xx} & q_{xy} & q_{xz} \\ q_{yx} & q_{yy} & q_{yz} \\ q_{zx} & q_{zy} & q_{zz} \end{bmatrix}$; $W =$

$\text{diag}(\frac{1}{L_1^2}, \frac{1}{L_2^2}, \frac{1}{L_3^2}, \frac{1}{L_4^2})$, 为权矩阵。

平差后的 p 点坐标为:

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} x_{p0} \\ y_{p0} \\ z_{p0} \end{bmatrix} + \begin{bmatrix} \delta x_p \\ \delta y_p \\ \delta z_p \end{bmatrix} \quad (5)$$

验后测距单位权中误差 σ_L 为:

$$\sigma_L = \sqrt{\frac{V^T W V}{4 - 3}} \quad (6)$$

根据式(4)和式(6)可得 p 点的点位精度为:

$$\sigma_p = \sigma_L \sqrt{q_{xx} + q_{yy} + q_{zz}} \quad (7)$$

p 点近似坐标是式(1)整体平方后, 式(1b)~式(1d)分别与式(1a)相减得:

$$B \begin{bmatrix} x_{p0} & y_{p0} & z_{p0} \end{bmatrix}^T = b \quad (8)$$

$$B = \begin{bmatrix} x_{T2} & 0 & 0 \\ x_{T3} & y_{T3} & 0 \\ x_{T4} & y_{T4} & z_{T4} \end{bmatrix}$$

$$b = \frac{1}{2} \begin{bmatrix} L_1^2 - L_2^2 + x_{T2}^2 \\ L_1^2 - L_3^2 + x_{T3}^2 + y_{T3}^2 \\ L_1^2 - L_4^2 + x_{T4}^2 + y_{T4}^2 + z_{T4}^2 \end{bmatrix}$$

最终 p 点近似坐标为:

$$x_{p0} = [x_{p0} \ y_{p0} \ z_{p0}]^T = B^{-1} b \quad (9)$$

2 系统参数标定与布局优化

2.1 无定向点系统参数标定法

根据激光测距多边交会原理, 需求解各测站在全局坐标系下的坐标, 该求解过程称为系统参数标定^[18]。系统参数标定方法有坐标转换法^[19]、多目标点法^[18]。但上述方法需布设若干定向点, 且系统参数标定精度与定向点分布有关。若两测站间的距离已知, 则可建立无定向点系统参数标定法。根据图 1 全局坐标系定义, 系统参数满足如下几何关系:

$$x_{T1} = y_{T1} = z_{T1} = y_{T2} = z_{T2} = z_{T3} = 0 \quad (10a)$$

$$x_{T2}^2 = d_{21}^2 \quad (10b)$$

$$x_{T3}^2 + y_{T3}^2 = d_{31}^2 \quad (10c)$$

$$x_{T4}^2 + y_{T4}^2 + z_{T4}^2 = d_{41}^2 \quad (10d)$$

$$(x_{T3} - x_{T2})^2 + y_{T3}^2 = d_{32}^2 \quad (10e)$$

$$(x_{T4} - x_{T2})^2 + y_{T4}^2 + z_{T4}^2 = d_{42}^2 \quad (10f)$$

$$(x_{T4} - x_{T3})^2 + (y_{T4} - y_{T3})^2 + z_{T4}^2 = d_{43}^2 \quad (10g)$$

式中, T_i, T_j 表示激光跟踪仪; d_{ij} 表示 T_i 中心与 T_j 中心的距离; $x_{T2}, x_{T3}, y_{T3}, x_{T4}, y_{T4}, z_{T4}$ 为未知系统参数。

由式(10c)、式(10e)求解 T_3 坐标, 由式(10d)、式(10f)和式(10g)求解 T_4 坐标, 对应解为:

$$\begin{cases} x_{T2} = \pm d_{21} \\ x_{T3} = \frac{d_{32}^2 - d_{31}^2 - x_{T2}^2}{-2x_{T2}} \\ y_{T3} = \pm \sqrt{d_{31}^2 - x_{T3}^2} \\ x_{T4} = \frac{d_{42}^2 - d_{41}^2 - x_{T2}^2}{-2x_{T2}} \\ y_{T4} = \frac{d_{43}^2 - d_{41}^2 - d_{31}^2 + 2x_{T3}x_{T4}}{-2y_{T3}} \\ z_{T4} = \pm \sqrt{d_{41}^2 - x_{T4}^2 - y_{T4}^2} \end{cases} \quad (11)$$

式中, 系统参数正负号由全局坐标系定义取舍。

2.2 球心拟合的测站间距测量

测站原点是仪器坐标系的原点且无法直接测

量,由于仪器垂直度盘与水平度盘正交,跟踪仪测头部的运动轨迹是以测站原点为球心的球。在跟踪仪 T_j 的测头部固定夹具(见图2),当 T_j 转动时,另一台跟踪仪 T_i 对 T_j 夹具上的球形反射器测量坐标为 $(x_i, y_i, z_i), i=1, 2, \dots, n$, 根据球心拟合方程得到 T_j 原点在 T_i 仪器坐标系下坐标 (x_{j0}, y_{j0}, z_{j0}) , 从而得到两测站间距 $d_{ij} = \sqrt{x_{j0}^2 + y_{j0}^2 + z_{j0}^2}$ 。半径为 r 的球心拟合方程为:

$$(x_i - x_{j0})^2 + (y_i - y_{j0})^2 + (z_i - z_{j0})^2 = r^2 \quad (12)$$

式(12)的球心坐标最小二乘解见文献[20]。

2.3 跟踪仪多边组网布局优化

由式(4)、式(7)可知,多边交会测量精度与测



图2 跟踪仪的测头部固定夹具

Fig.2 Holder Fixed on Head of Laser Tracker

站布局相关。此外,文献[13-14]证实测站布局对多边交会精度影响显著。本文从无定向点系统参数标定误差、激光测距多边交会误差两方面优化测站布局。设两跟踪仪原点间距离测量误差为 σ_d , 式(11)求导得到无定向点系统参数标定误差为:

$$\begin{cases} \sigma_{x_{T2}} = \sqrt{\left(\frac{\partial x_{T2}}{\partial d_{21}}\right)^2} \sigma_{d_{21}}^2 = \sigma_{d_{21}} = \sigma_d \\ \sigma_{x_{T3}} = \sqrt{\left(\frac{\partial x_{T3}}{\partial d_{32}}\right)^2 \sigma_{d_{32}}^2 + \left(\frac{\partial x_{T3}}{\partial d_{31}}\right)^2 \sigma_{d_{31}}^2 + \left(\frac{\partial x_{T3}}{\partial d_{T2}}\right)^2 \sigma_{x_{T2}}^2} = \sqrt{\frac{d_{32}^2 + d_{31}^2}{x_{T2}^2} + \frac{1}{4} \left(\frac{d_{32}^2 - d_{31}^2}{x_{T2}^2} + 1 \right)} \sigma_d \\ \sigma_{y_{T3}} = \sqrt{\left(\frac{\partial y_{T3}}{\partial d_{31}}\right)^2 \sigma_{d_{31}}^2 + \left(\frac{\partial y_{T3}}{\partial x_{T3}}\right)^2 \sigma_{x_{T3}}^2} = \sqrt{\frac{d_{31}^2}{d_{31}^2 - x_{T3}^2} \sigma_{d_{31}}^2 + \frac{x_{T3}^2}{y_{T3}^2} \sigma_{x_{T3}}^2} \\ \sigma_{x_{T4}} = \sqrt{\left(\frac{\partial x_{T4}}{\partial d_{42}}\right)^2 \sigma_{d_{42}}^2 + \left(\frac{\partial x_{T4}}{\partial d_{41}}\right)^2 \sigma_{d_{41}}^2 + \left(\frac{\partial x_{T4}}{\partial x_{T2}}\right)^2 \sigma_{x_{T2}}^2} = \sqrt{\frac{d_{42}^2 + d_{41}^2}{x_{T2}^2} + \frac{1}{4} \left(\frac{d_{42}^2 - d_{41}^2}{x_{T2}^2} + 1 \right)} \sigma_d \\ \sigma_{y_{T4}} = \sqrt{\left(\frac{\partial y_{T4}}{\partial d_{43}}\right)^2 \sigma_{d_{43}}^2 + \left(\frac{\partial y_{T4}}{\partial d_{41}}\right)^2 \sigma_{d_{41}}^2 + \left(\frac{\partial y_{T4}}{\partial d_{31}}\right)^2 \sigma_{d_{31}}^2 + \left(\frac{\partial y_{T4}}{\partial x_{T3}}\right)^2 \sigma_{x_{T3}}^2 + \left(\frac{\partial y_{T4}}{\partial x_{T4}}\right)^2 \sigma_{x_{T4}}^2 + \left(\frac{\partial y_{T4}}{\partial y_{T3}}\right)^2 \sigma_{y_{T3}}^2} = \\ \sqrt{\frac{d_{43}^2}{y_{T3}^2} \sigma_{d_{43}}^2 + \frac{d_{41}^2}{y_{T3}^2} \sigma_{d_{41}}^2 + \frac{d_{31}^2}{y_{T3}^2} \sigma_{d_{31}}^2 + \frac{x_{T4}^2}{y_{T3}^2} \sigma_{x_{T3}}^2 + \frac{x_{T3}^2}{y_{T3}^2} \sigma_{x_{T4}}^2 + \frac{1}{4} \left(\frac{d_{43}^2 - d_{41}^2 - d_{31}^2 + 2x_{T3}x_{T4}}{y_{T3}^2} \right)^2 \sigma_{y_{T3}}^2} \\ \sigma_{z_{T4}} = \sqrt{\left(\frac{\partial z_{T4}}{\partial d_{41}}\right)^2 \sigma_{d_{41}}^2 + \left(\frac{\partial z_{T4}}{\partial x_{T4}}\right)^2 \sigma_{x_{T4}}^2 + \left(\frac{\partial z_{T4}}{\partial y_{T4}}\right)^2 \sigma_{y_{T4}}^2} = \sqrt{\frac{d_{41}^2}{z_{T4}^2} \sigma_{d_{41}}^2 + \frac{x_{T4}^2}{z_{T4}^2} \sigma_{x_{T4}}^2 + \frac{y_{T4}^2}{z_{T4}^2} \sigma_{y_{T4}}^2} \end{cases} \quad (13)$$

由式(13)可知, $\sigma_{x_{T2}}$ 仅与 σ_d 有关,其他系统参数中误差包含复杂的误差传递系数。为消去 $\sigma_{x_{T4}}$ 、 $\sigma_{y_{T3}}$ 的误差传递系数,使得其误差最小,则有 $x_{T3} = x_{T4} = y_{T4} = 0$,代入式(13)可得:

$$\begin{cases} \sigma_{x_{T2}} = \sigma_{d_{21}} = \sigma_d \\ \sigma_{x_{T3}} = \sqrt{2 \left(1 + \frac{y_{T3}^2}{x_{T2}^2} \right)} \sigma_d \\ \sigma_{y_{T3}} = \sigma_{d_{31}} = \sigma_d \\ \sigma_{x_{T4}} = \sqrt{2 \left(1 + \frac{z_{T4}^2}{x_{T2}^2} \right)} \sigma_d \\ \sigma_{y_{T4}} = \sqrt{2 \left(1 + \frac{z_{T4}^2}{y_{T3}^2} \right)} \sigma_d \\ \sigma_{z_{T4}} = \sigma_{d_{41}} = \sigma_d \end{cases} \quad (14)$$

由式(14)可知, $\sigma_{x_{T2}}$ 、 $\sigma_{y_{T3}}$ 、 $\sigma_{z_{T4}}$ 仅与 σ_d 有关。若使 $\sigma_{x_{T3}}$ 最小化,则 $y_{T3}=0$,但与 $\sigma_{y_{T4}}$ 矛盾;若 $\sigma_{y_{T4}}$ 、 $\sigma_{x_{T4}}$ 达到最小值,则 $z_{T4}=0$,但导致 $T_4=(0,0,0)$ 与 T_1 重合。

综上所述,为减小系统参数标定误差,令测站 T_1 位于坐标系原点 O , T_2 、 T_3 、 T_4 分别在 x 轴、 y 轴和 z 轴上, T_1 、 T_2 、 T_3 和 T_4 构成直角三棱锥,如图3所示。

为确定图3中 x_{T2} 、 y_{T3} 、 z_{T4} 的取值,从激光测距多边交会测量误差分析,将式(8)中矩阵 b 改写为:

$$b = s + \frac{1}{2} f(B \cdot B) \quad (15)$$

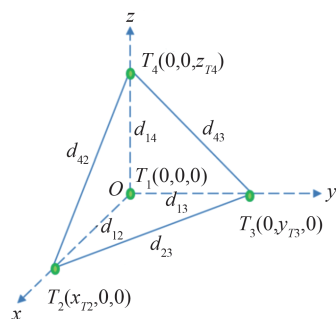


图3 直角三棱锥布局

Fig.3 Layout of Rectangular Triangular Pyramid

式中, $s = \frac{1}{2} \begin{bmatrix} L_1^2 - L_2^2 \\ L_1^2 - L_3^2 \\ L_1^2 - L_4^2 \end{bmatrix}$; $f(B \cdot B)$ 表示 $B \cdot B$ 的元素

按行求和; $B \cdot B$ 表示 B 中每个元素求平方。

由式(8)可知, B 与测站布局相关, 且 B 已经由无定向点系统参数标定得到, 故式(8)仅考虑测站至测点距离的扰动 δs 影响, 结合式(15), 式(8)中测点坐标扰动误差 δx_p 满足:

$$B(x_{p0} + \delta x_p) = (s + \delta s) + \frac{1}{2} f(B \cdot B) \quad (16)$$

将式(8)代入式(16), 得 $B\delta x_p = \delta s$, 则 $\delta x_p = B^{-1}\delta s$, 由矩阵范数相容性原理有 $\|\delta x_p\| = \|B^{-1}\delta s\| \leq \|B^{-1}\| \cdot \|\delta s\|$ 。式(8)两边取范数可得 $\|b\| = \|Bx_{p0}\| \leq \|B\| \cdot \|x_{p0}\|$, 因为 $\|x_{p0}\| \neq 0$ 且 $\|b\| \neq 0$, 于是有:

$$\frac{\|\delta x_p\|}{\|x_{p0}\|} \leq \|B^{-1}\| \cdot \|B\| \cdot \frac{\|\delta s\|}{\|b\|} \quad (17)$$

由式(17)可知, δx_p 与 $\|B^{-1}\| \cdot \|B\|$ 相关, 当 $\|B^{-1}\| \cdot \|B\|$ 取得最小值时, 对应测站布局最佳。在图3坐标系下, $B = \text{diag}(x_{T2}, y_{T3}, z_{T4})$, 那么。

$$\begin{aligned} \|B^{-1}\| \cdot \|B\| &= \sqrt{\frac{1}{x_{T2}^2} + \frac{1}{y_{T3}^2} + \frac{1}{z_{T4}^2}} \sqrt{x_{T2}^2 + y_{T3}^2 + z_{T4}^2} \\ &= \sqrt{3 + \frac{y_{T3}^2}{x_{T2}^2} + \frac{z_{T4}^2}{y_{T3}^2} + \frac{x_{T2}^2}{z_{T4}^2} + \frac{z_{T4}^2}{x_{T2}^2} + \frac{y_{T3}^2}{z_{T4}^2} + \frac{x_{T2}^2}{y_{T3}^2}} \end{aligned}$$

由不等式定理可得 $\|B^{-1}\| \cdot \|B\| \geq 3$, 且当 $x_{T2} = y_{T3} = z_{T4}$ 成立时, 测站布局为直角正三棱锥。

3 多边交会测量精度分析

3.1 系统参数标定精度分析

根据无定向点系统参数标定原理, 两测站原点间距是通过球心拟合得到, 而球心拟合精度与测点数量、测点覆盖球面范围相关。选用跟踪仪

的 T-probe(图4)扩展功能^[21], 搭配 1.5 英寸半球探针实现测点覆盖整个球面。测站按直角正三棱锥布局 ($x_{T2} = y_{T3} = z_{T4} = 5$), 基于蒙特卡罗仿真原理^[2, 22], 通过 10 000 次仿真得到球心拟合中误差与测点数和测点覆盖球面范围的关系, 分别如图5和图6所示。跟踪仪测距按照标准正态分布 $N(0, 0.5 \mu\text{m/m})$ 、测角按照标准正态分布 $N(0, 1.5'')$ 分别添加随机误差。



图4 跟踪仪的 T-probe

Fig.4 T-probe of Laser Tracker

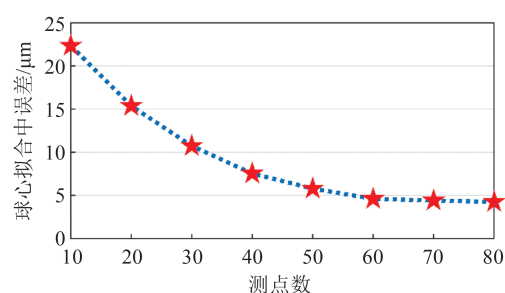


图5 球心拟合中误差与测点数的关系

Fig.5 Relation of Root Mean Square Error of Spherical Center Fitting and Number of Points

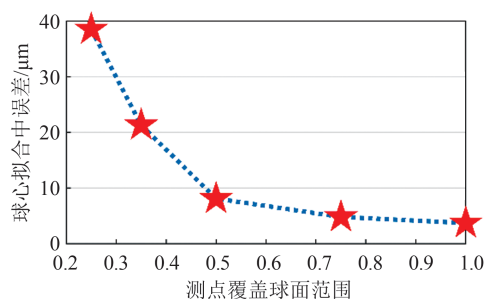


图6 球心拟合中误差与测点覆盖球面范围的关系

Fig.6 Relation of Root Mean Square Error of Spherical Center Fitting and Measuring Coverage

由图5可知, 当测点覆盖整个球面时, 球心拟合中误差与测点数成正相关; 当测点数为 50, 球心拟合中误差约为 0.005 mm, 此后随着测点数增加, 球心拟合中误差几乎不变。由图6可知, 固定测点数为 50, 球心拟合中误差与测点覆盖球面范围成正相关; 当测点覆盖球面的 3/4 时, 球心原点

拟合中误差接近0.005 mm,此后再增加测点覆盖球面范围,球心拟合中误差几乎不变。综上可知,测点数为50且测点覆盖球面范围为3/4,球心拟合中误差可达到0.005 mm。

球心拟合得到测站原点坐标,继而得到测站间的距离,由式(11)即可标定系统参数。基于蒙特卡罗方法得到系统参数标定中误差如图7所示。

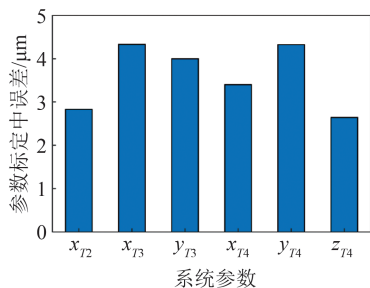


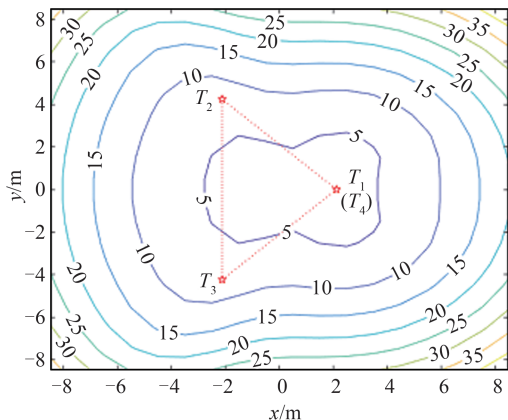
图7 系统参数标定中误差

Fig.7 Root Mean Square Error of System Parameters Calibration

由图7可知,测站 T_2 、 T_3 、 T_4 参数标定的点位中误差分别为0.002 8 mm、0.005 5 mm、0.006 4 mm。 T_4 中误差最大,其次是 T_3 ,最后是 T_2 ,这与式(14)一致。取参数标定最大点位中误差作为系统参数标定中误差,即 $u_T=0.006 4$ mm。

3.2 多边交会测点精度分析

4台激光跟踪仪按直角正三棱锥布局($x_{T2}=y_{T3}=z_{T4}=5$),假设各测站原点坐标已知,添加测站至测点距离的标准正态分布随机误差 $N(0, 0.5 \mu\text{m}/\text{m})$ 。根据多边交会原理,通过10 000次仿真计算得到多边交会测点中误差分布,如图8所示。



注:中误差等值线单位为 μm

图8 多边交会测点中误差分布

Fig.8 Distribution of Root Mean Square Error of Points Calculated by Multilateral Method

由图8可知,测点在测站构成的直角正三棱锥内,其点位中误差小于0.005 mm;测点在直角正三棱锥外侧,其点位中误差以同心环向外增大且增速加快。因此多边交会测点时,测点应在测站构成的直角三棱锥内侧。在5 m范围内,多边交会测点中误差 $u_p=0.005$ mm。

4 实验验证

4.1 标准尺的长度测量实验

本次实验采用的标准器与调节装置如图9所示,图9(a)为标准尺,是由热膨胀系数较低的碳纤维制成,其两端球形反射器的间距已知(精度优于0.003 mm);图9(b)为坐标测量实验的四面体标准器;图9(c)为标准尺的调节装置。

4台激光跟踪仪按直角正三棱锥布局($x_{T2}=y_{T3}=z_{T4}=5$)构成5 m \times 5 m测区,标准尺固定在调节装置,使其在测区的横向、纵向、垂向、倾斜4个方向均有分布。系统参数标定后,按照多边交会原理测得标准尺上球形反射器的坐标并反算出标准尺的长度,标准尺的长度计算值与参考值的测量偏差 ΔL 如图10所示。

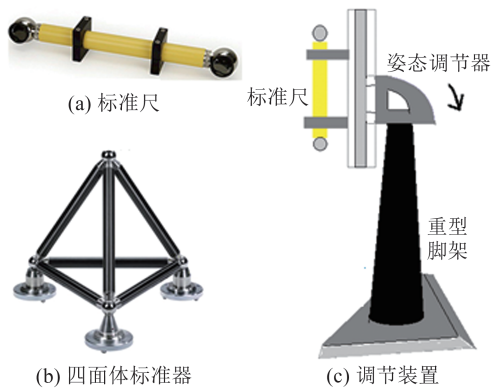


图9 标准器与调节装置

Fig.9 Calibrator and Adjusting Device

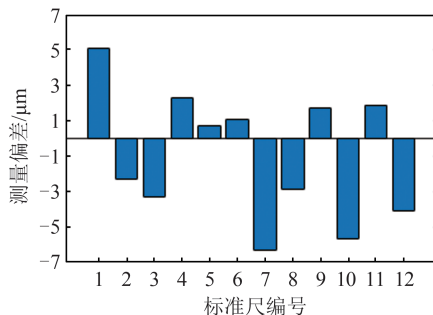


图10 标准尺的长度计算值与参考值的偏差

Fig.10 Deviation of Calculations of the Standard Length from the Reference Data

由图 10 可知,标准尺的长度计算值与参考值最大偏差为 0.005 1 mm,最小偏差为−0.006 3 mm,平均偏差为−0.001 mm。在 5 m×5 m 范围内,由标准尺的长度偏差值得到多边交会的测距中误差为 0.003 6 mm。

采用 $E_n=|\Delta L|/\sqrt{u_L^2+u_{L0}^2}$ 检验标准尺的长度计算值与参考值的一致性。其中,参考值中误

差 $u_L=0.003$ mm;计算值中误差 $u_{L0}=\sqrt{u_T^2+u_p^2}=0.008$ 1 mm; u_T 为系统参数标定中误差,由 §3.1 得到; u_p 为多边交会测点中误差,由 §3.2 得到。标准尺长度测量偏差检验结果见表 1。由表 1 可以看出,12 次计算的 E_n 值均小于 1,表明多边交会的标准尺长度测量值与参考值的一致性通过。

表 1 标准尺长度测量偏差检验
Tab.1 Length Deviation Testing of Standard Length Measurement

测长检验	标准尺的分布方向											
	横向			纵向			垂向			倾斜		
$\Delta L(\mu\text{m})$	5.1	−2.3	−3.3	2.3	0.7	1.1	−6.3	−2.9	1.7	−5.7	1.9	−4.1
E_n	0.5	0.2	0.3	0.3	0.1	0.1	0.7	0.3	0.2	0.7	0.2	0.5

4.2 四面体标准器坐标测量实验

四面体标准器由稳定性极高的碳纤维杆组成(图 9(b)),其 4 个顶点处的球形反射器坐标是由三坐标测量机测得,且精度优于 0.002 mm 并作为参考值。4 台激光跟踪仪按直角正三棱锥

布局($x_{T2}=y_{T3}=z_{T4}=5$),系统参数标定后按照多边交会原理计算四面体上球形反射器坐标,并将坐标计算结果转换为三坐标测量机测量结果,两者的坐标偏差(Δx 、 Δy 、 Δz)与点位偏差 ΔS 分布见表 2。

表 2 多边交会与三坐标测量机的坐标测量偏差分布 /mm
Tab.2 Coordinate Deviation of Multilateral Method and Coordinate Measurement Machine /mm

点名	三坐标测量机坐标测量值	Δx	Δy	Δz	ΔS
P1	(0,0,0)	−0.001 2	−0.002 8	−0.005 2	0.006 0
P2	(800.058 0,0,0)	0.001 0	−0.001 5	−0.004 8	0.005 1
P3	(500.053 5,624.049 2,0)	−0.001 0	0.001 6	−0.005 6	0.005 9
P4	(500.016 4,285.583 6,555.801 2)	0.001 9	0.001 2	−0.003 1	0.003 8

由表 2 可知,5 m 范围内多边交会测得点位偏差 ΔS 不大于 0.0060 mm,由 P1~P4 点位偏差得到多边交会的单点测量中误差为 0.005 3 mm,远小于单站 5 m 范围内测点标称中误差 0.045 mm。

5 结 语

基于球心拟合的无定向点参数标定法,可直接计算各测站在全局坐标系下坐标。基于无定向点参数标定模型推导出四路激光测距交会的最佳测站布局为直角正三棱锥,从而优化多边交会测量精度。通过标准尺、四面体标准器分别验证优化后多边交会的测距精度以及测点精度。实验结果表明,在 5 m 范围内,优化后多边交会的测距中误差优于 0.003 6 mm,测点中误差达到 0.005 3 mm。

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Measurement Accuracy Analysis of Multilateral Method Based on Optimal Layout with Non-directional Point Calibration

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Abstract: Objectives: The measuring accuracy of the multilateral method is affected by the station layout and the calibration errors of system parameters. The optimal station layout and new system parameters calibration method are investigated for optimizing the multilateral method. **Methods:** A spherical center fitting method is introduced to calibrate station coordinates without enhanced reference points. Besides, according to the model error of the multilateral method, the optimal station layout is an equilateral rectangular triangular pyramid. **Results:** In the range of 5 m, the root mean square error (RMSE) of system parameters calibration based on spherical center fitting method is less than 0.006 4 mm, and the RMSE of coordinate measurement of point inside the optimal station layout is less than 0.005 mm. Within the survey region of 5 m, the RMSE of length measurement is less than 0.003 6 mm using optimized multilateral method, and the RMSE of coordinate measurement of optimized multilateral method is less than 0.005 3 mm. **Conclusions:** The optimized multilateral method can achieve high-precision coordinate measurement in a small region. The large-scale 3D coordinate measurement will be studied later using multilateral method.

Key words: non-directional point calibration; optimization of station layout; multilateral method; laser tracker

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